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PASSAGE OF SOLID PARTICLES THROUGH ROTARY CYLINDRICAL KILNS

BY

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and OLIVER C. RALSTON

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PASSAGE OF SOLID PARTICLES THROUGH ROTARY CYLINDRICAL KILNS

By J. D. Sullivan, Charles G. Maier, and Oliver C. Ralston

INTRODUCTION

In the course of its efforts to increase efficiency in the mineral industries the Bureau of Mines is investigating the basic principles involved in the construction and operation of devices for the treatment of ores and minerals. This paper presents the results of a study of the flow of material through rotary kilns of the type used in the manufacture of Portland cement clinker. A rotary kiln is a hollow cylinder, the length several times the diameter, which is placed with its axis inclined a few degrees from the horizontal, so that material fed into its upper end advances through it and discharges at the lower end.

The object of this study was to determine the factors that influence the rate of progress of crushed materials through a kiln. There is distinct need for such data in the design and construction of kilns for drying, roasting, calcining, nodulizing, or otherwise treating crushed solid materials. Hitherto the practice in the design of such kilns has been largely to copy from kilns already in use the dimensions that give the properties most nearly satisfactory. The writers wished to develop a "kiln formula" that would fit all the conditions studied and remove guesswork from the design of rotary kilns.

As a result of this study it was possible to assemble all the facts noted into a single equation which expresses the time of passage of solid materials through a rotating cylindrical kiln of any of the sizes usually desired, when the kiln is operating under conditions that approximate those of actual practice. This kiln formula is entirely empirical but usually predicts within 2 or 3 per cent and is seldom more than 6 per cent in error. It is as follows:

$$t = \frac{1.77 l \sqrt{\theta}}{pdn} \times \text{factor}$$

where t = time of stay of the material in the kiln in minutes.
l = length of the kiln in feet.
p = slope or pitch of the kiln in degrees.
d = diameter of the kiln in feet.
n = revolutions per minute.
θ = angle of repose, in degrees, of material in the dry condition.

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Other symbols which will be used in succeeding formulas are as follows:

v_f = volume of feed per hour.

v = volume of the kiln; v_f and v are to be expressed in the same units; for example, in cubic centimeters or cubic feet.

h = height of the constriction [$=\frac{1}{2}(d^\circ - d')$].

d' = diameter of constriction opening.

d° = diameter of the kiln; h, d', and d° must be expressed in the same units. $d^\circ = d$, when d° is expressed in feet.

The "factor" is a correction for nose rings or constrictions. The value of the factor is 1 for simple unobstructed rotary cylindrical kilns.

The steps by which the formula was obtained follow. From the wealth of data collected only enough tables and figures are given to establish the relationships.

The mathematical principles developed in this report should find useful application in the design of all types of rotary-kiln furnaces, cylindrical driers, and cylindrical feeders, not only in the mining and mineral industries, but in other industries employing similar machinery.

EXPERIMENTAL KILNS USED

The general principles and the mathematical formula were developed by study of a small laboratory kiln built on such a scale that a 1-inch dimension represented roughly about 1 foot of a full-size kiln. Later the kiln formula developed as a result of this research was checked against a variety of operating kilns. The experimental kiln was electrically driven through a friction disk with which the rate of revolution could be varied at will. The slope could be changed by means of a worm-gear jack under one end. The kiln was 7 feet long, and a series of shells 3, 6, 11½, and 19¾ inches in diameter was used in alternation for the study of diameter effects. A 5-foot kiln, 3¾ inches in diameter and electrically heated, was also used. This equipment in all amounted to five separate kilns, 3, 3¾, 6, 11½, and 19¾ inches in diameter. The interior of the shells was made rough by sizing it with glue or varnish and coating it with sand. The discharge end was open, but at the feed end was a diaphragm having an opening just large enough for the feed pipe from a mechanical feeder to be slipped through readily. This feeding device was capable of delivering varying amounts of feed.

Figure 1 is a photograph of the electrically heated kiln and shows the mechanism, including friction disk drive, gear for adjusting slope, variable feeder, collecting rings for electric heating current, tires, and bearings.

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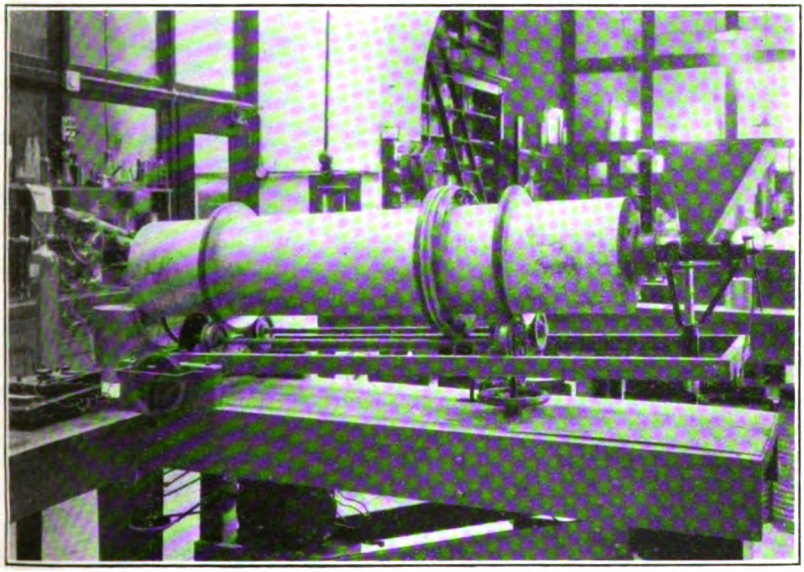

FIGURE 1.—Experimental rotary kiln

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METHOD OF STUDYING THE PROBLEM

The chief problem attacked was determination of the time intervals required for a given material to pass through the kiln under varying conditions; in other words, the rate of advance per foot. The time of stay was determined as follows: The kiln was fed at a uniform rate until the bed in the whole kiln had come to a condition of equilibrium and the rate of discharge was constant; then the weight of material discharged in a given time (usually 15 minutes) was found. The kiln was stopped, the feeding device disconnected, and the material remaining in the kiln discharged mechanically. From these two weighings the duration of stay is at once determined. Let x = the weight of material discharged in y minutes, and let z = the weight of material remaining in the kiln; then $z \times y/x$ = time of passage of the average particle of material through the kiln.

EFFECT OF VARIABLES ON TIME OF PASSAGE

The time of passage was studied as a function of each of the following variables: Slope of kiln, rate of feed, r. p. m., length of kiln, diameter of kiln, angle of repose of the material, and the size of the material. Each variable was investigated separately; all other things were constant except the one being investigated. For example, in determining the effect of the slope of the kiln a given material was used and the kiln rotated at a constant speed, with a constant rate of feed, and in a given kiln the only thing changed was the slope of the kiln itself.

EFFECT OF SLOPE OF KILN

To determine the effect of the slope of the kiln, different materials (a partial list is included under the subhead "Effect of angle of repose") were passed through the kiln, all other conditions but the slope being kept constant. The investigation has shown that the time of passage is inversely proportional to the pitch or slope of the kiln in degrees; that is, the time of passage at 6° is one-half of that at 3°, other conditions being constant. Tables 1 and 2 and Figures 2 and 3 will show this. Ottawa standard sand, 20 to 30 mesh size, was used in many of these tests in order to allow check by other investigators. This sand is in common use by civil engineers for testing cement in standard forms.

TABLE 1.—*Ottawa standard sand in 6-inch kiln at 2.5 r. p. m.*

Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes
1	3,585	39.4	4	4,135	14.2
2	3,821	28.2	5	4,300	10.9
3	4,105	18.3	6	4,280	9.1

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TABLE 2.—*Ottawa standard sand in 6-inch kiln at 2.5 r. p. m.*

Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes
1	342	51.0	4	374	13.3
2	369	26.4	5	368	10.9
3	396	17.6	6	369	8.9

In Figure 2 there is a dropping-off effect at 1° which might tend to indicate that the conclusion that the time is inversely proportional to the pitch is not true at low angles. It will be noted, however, that in Figure 3, which represents results with the same material at a lower rate of feed, the apparent discrepancy is not very noticeable.

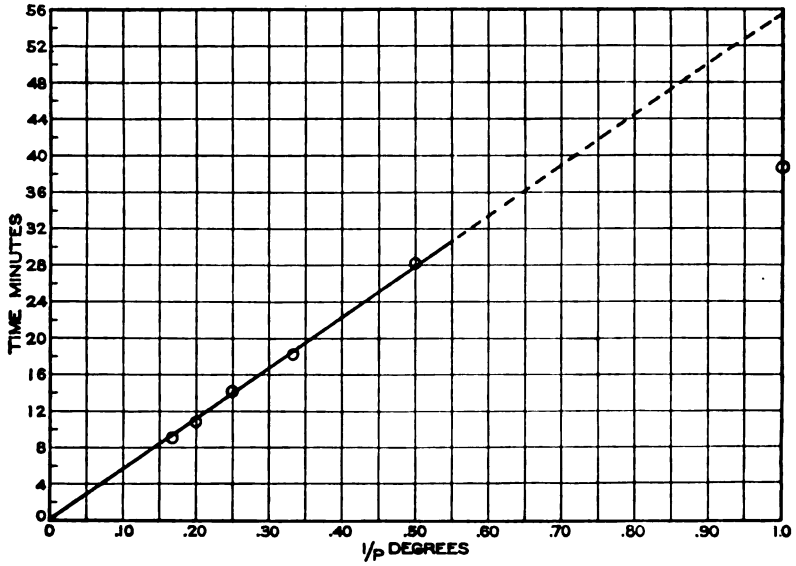

FIGURE 2.—Ottawa standard sand in 6-inch kiln; 2.5 r. p. m.; feed, 16,000 c. c. per hour

To determine why this effect is apparent when the bed is deep and not when it is thin, a device was made for measuring the depth of bed of material along the entire length of the kiln while the kiln was in motion. This device consisted of a cam-shaped piece of sheet metal set on a steel rod which was accurately centered on the axis of the kiln and placed in bearings so that the cam could be placed at any point along the axis of the kiln and then rotated until it barely touched the surface of the bed of material passing through the kiln. Previous calibration allowed the depth of the bed of material to be read in this way.

DATA ON DEPTH OF BED

Tables 3 and 4 and Figures 4 and 5, in which the depth of the bed is plotted on an exaggerated scale, contain data corresponding to Tables 1 and 2 and Figures 2 and 3.

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TABLE 3.—*Depth of bed of Ottawa standard sand in 6-inch kiln at 2.5 r. p. m. with rate of feed approximately 4,000 c. c. per 15 minutes*

Distance from discharge end of kiln, inches	Depth of bed, millimeters					
	1° slope	2° slope	3° slope	4° slope	5° slope	6° slope
6	23.6	23.9	20.8	20.8	19.2	17.5
12	34.6	31.8	26.1	25.0	20.8	19.2
18	38.3	35.5	30.4	26.1	23.9	20.8
24	42.1	39.5	33.0	28.9	25.0	22.4
30	45.2	41.0	34.3	30.4	26.1	23.9
36	48.0	42.4	35.5	30.4	26.1	23.9
42	50.9	44.4	35.5	30.4	26.1	23.9
48	54.1	45.3	35.5	30.4	26.1	23.9
54	57.0	46.5	35.5	28.9	26.1	23.9
60	57.0	46.5	35.5	28.9	26.1	23.9
66	60.5	46.5	34.3	27.6	25.0	22.4
72	60.5	46.5	34.3	26.1	24.2	22.4
78	60.5	46.5	33.0	26.1	23.9	20.8

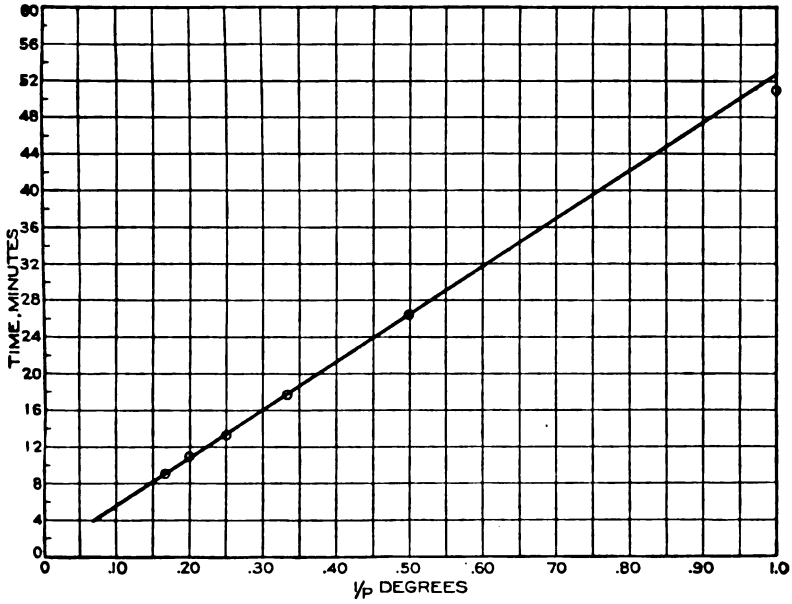

FIGURE 3.—Ottawa standard sand in 6-inch kiln; 2.5 r. p. m.; feed, 1,500 c. c. per hour

TABLE 4.—*Depth of bed of Ottawa standard sand in 6-inch kiln at 2.5 r. p. m. with rate of feed approximately 375 c. c. per 15 minutes*

Distance from discharge end of kiln, inches	Depth of bed, millimeters			
	1° slope	2° slope	3° slope	4° slope
6	9.4	7.2	7.9	4.6
12	10.6	9.4	9.4	6.4
18	14.0	10.6	10.6	7.0
24	14.9	11.0	10.6	7.5
30	14.9	11.7	10.6	7.9
36	14.9	11.7	10.6	7.9
42	14.9	11.7	11.7	7.9
48	14.9	11.7	11.7	7.9
54	14.9	11.7	10.6	7.9
60	14.9	10.6	10.6	7.9
66	14.9	10.6	10.6	7.5
72	14.9	9.4	7.9	7.0
78	14.0	9.4	7.9	7.0

89793°—27——2

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EXPLANATION OF DIFFERENT RESULTS FOR THICK AND THIN BEDS

Comparison of Figures 4 and 5 shows why this apparent discrepancy is noted at 1° for the deep bed. There is a "piling up" at the feed end of the kiln when the bed is deep, which does not occur when the bed is thin. Inasmuch as there is a piling-up effect, the material is

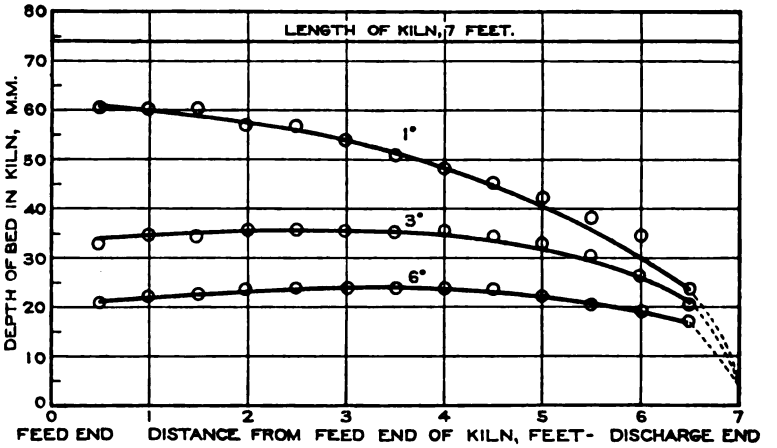

FIGURE 4.—Depth of bed of Ottawa standard sand at different points in kiln; 2.5 r. p. m.; slope, 1 to 6°; feed, 16,000 c. c. per hour

actually traveling down a steeper slope than that indicated by the slope of the kiln, consequently the material should be expected to pass through the kiln faster than would be required by the relationship $t \propto 1/p$. In a thin bed this piling-up effect is nearly negligible, and the plot of t v. $1/p$ includes very nearly all the points on a

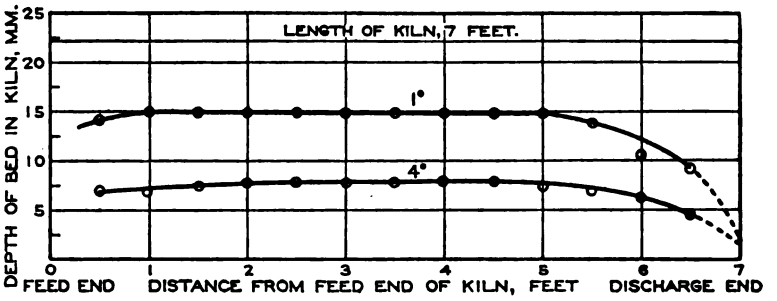

FIGURE 5.—Depth of bed of Ottawa standard sand at different points in kiln; slope, 1 to 4°; feed, 1,500 c. c. per hour

straight line, even at 1°. The reader should note that this piling up is a special effect found only at one end of a kiln and may be negligible in commercial kilns, as their length is great in comparison to their diameter. It is doubtful if this effect is at all noticeable in a kiln 100 feet long. Furthermore, in actual practice, most kilns

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run at an angle much greater than 1°, usually 0.75 inch per foot (3.57°) or 0.5 inch per foot (2.39°).

DATA ON SEVERAL MATERIALS

Tables 5 and 6 further show that this relationship ($t \propto 1/p$ or $t \times p = \text{constant}$) is true:

TABLE 5.—*20 to 30 mesh quartz in 6-inch kiln at 2.5 r. p. m.*

Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	$t \times p$	Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	$t \times p$
2	3,221	29.5	59.0	5	3,280	12.2	61.0
3	3,240	20.6	61.8	6	3,365	10.0	60.0
4	3,320	15.4	61.6				

TABLE 6.—*20 to 30 mesh copper smelting slag in 6-inch kiln at 2.5 r. p. m.*

Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	$t \times p$	Angle, degrees	Feed in 15 minutes, c. c.	Time in kiln, minutes	$t \times p$
2	3,255	30.4	60.8	5	3,462	12.5	62.5
3	3,509	21.2	63.6	6	3,280	10.1	60.6
4	3,490	15.3	61.2	8	4,430	7.5	60.0

An abundance of data was collected, but those given will suffice. References to tables and curves given in later parts of this discussion, however, will also tend to prove that time of the passage through the kiln varies inversely as the pitch or slope expressed in degrees.

USE OF ANGULAR MEASURE

The use of angular measure in place of trigonometric functions in the derived kiln formula may be explained here. For the slope or pitch of the kiln p, the angles studied usually ranged from 1 to 8°, and in this range the angle p and sin p or tan p are directly proportional to each other for the first three significant figures of the ratio sin p/p or tan p/p, which is as close as could be measured. It is therefore more convenient to use the circular measure and eliminate the trigonometric function. For the angle of repose of the material, θ, it was found empirically that the use of angular measure in the formula gave more exact figures than the use of any trigonometric function. Dimensionally the kiln equation is correct.

$$t = \frac{1.77\, l\, \sqrt{\theta}}{pdn}$$

$$\text{sec.} = \frac{\text{ft.}}{\text{ft.} \times 1/\text{sec.}}$$

$$\text{sec.} = \text{sec.}$$

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However, at best the kiln equation is empirical and the simplest form in which it was possible to express the equation and still retain accuracy has been chosen.

EFFECT OF RATE OF FEED

The variable, rate of feed, was studied, all other factors being kept constant. As the kiln was equipped with a mechanical feeding device, the rate of feed could be varied over rather wide limits. The following tables show the conclusion to be drawn from this study, namely, that the time of passage through the kiln is independent of the rate of feed.

TABLE 7.—*Effect of rate of feed of 30 to 50 mesh crushed quartzite in 3¾-inch kiln at angle of 8°*

Feed in 15 minutes, c. c.	Time in kiln, minutes	Feed in 15 minutes, c. c.	Time in kiln, minutes
135. 1	11. 5	427. 6	11. 0
200. 0	11. 4	465. 5	11. 2
265. 8	11. 1	510. 2	11. 4
349. 5	11. 3		

TABLE 8.—*Effect of rate of feed of 50 to 80 mesh crushed quartzite in 3¾-inch kiln at angle of 3°*

Feed in 15 minutes, c. c.	Time in kiln, minutes	Feed in 15 minutes, c. c.	Time in kiln, minutes
123. 0	32. 2	244. 0	36. 2
170. 8	32. 3	297. 2	33. 8
192. 6	35. 2	308. 8	32. 6
204. 6	37. 3		

Also compare the corresponding figures in Tables 1 and 2. Here, except at 1° and slightly at 2°, where abnormal results already accounted for exist, the time of passage is virtually the same at corresponding slopes of the kiln, although the feed is over 10 times as great in one instance as in the other. Table 8 was chosen because it shows about as wide a variation of results as any set studied but still indicates a very definite relationship. Tests showed that when the rate of feed was so high that the maximum amount of material which the kiln could handle was introduced, the time of passage increased slightly, but never more than about 5 or 6 per cent.

Investigation of the effect of rate of feed shows that over a very wide range, at least, the time of passage is independent of the rate of feed. The only effect of a higher rate of feed is to give a deeper bed of material in the kiln. Of course, per unit weight of material, the surface area exposed to kiln gases is lessened when the rate of feed is increased.

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EFFECT OF RATE OF ROTATION

The effect of the rate of rotation of the kiln was studied by feeding a given material through the kiln under given conditions and then repeating with the kiln revolving at a different rate. This investigation, as the following tables show, proved that the time of passage is inversely proportional to the speed of rotation. Table 9 is based on the assumption that $t \propto 1/n$.

TABLE 9.—*Comparison of time of passage of Ottawa standard sand in 3¾-inch kiln at 2.0 and 2.8 r. p. m.*

Angle, degrees	Time of passage at 2.8 r. p. m., minutes	Time of passage at 2.0 r. p. m., minutes		Angle, degrees	Time of passage at 2.8 r. p. m., minutes	Time of passage at 2.0 r. p. m., minutes	
		Observed	Calculated			Observed	Calculated
3	19.5	27.0	27.4	8	7.4	10.0	10.4
4	14.6	20.2	20.4	10	6.0	8.0	8.4
5	11.7	16.0	16.4	12	5.0	6.6	7.0
6	9.8	13.4	13.7				

Table 10 gives the data comparing runs made with 50 to 80 mesh crushed quartzite at 1 and 2 r. p. m.

TABLE 10.—*Runs with 50 to 80 mesh quartzite*

Angle, degrees	Time of passage at 1 r. p. m., minutes	Time of passage at 2 r. p. m., minutes		Angle, degrees	Time of passage at 1 r. p. m., minutes	Time of passage at 2 r. p. m., minutes	
		Observed	Calculated			Observed	Calculated
3	57.2	27.9	28.6	8	21.2	11.1	10.6
4	42.8	22.7	21.4	10	16.8	8.7	8.4
5	34.2	18.0	17.1	12	13.8	7.1	6.9
6	28.4	14.5	14.2				

Table 11 gives a comparison of 20 to 30 mesh quartz in the 11½-inch kiln at 1 and 1.6 r. p. m.

TABLE 11.—*Comparison of 20 to 30 mesh quartz at 1 and 1.6 r. p. m.*

Angle, degrees	Time of passage at 1 r. p. m., minutes	Time of passage at 1.6 r. p. m., minutes		Angle, degrees	Time of passage at 1 r. p. m., minutes	Time of passage at 1.6 r. p. m., minutes	
		Observed	Calculated			Observed	Calculated
2	33.2	20.5	20.7	5	15.7	9.95	9.8
3	25.2	16.5	15.7	6	12.4	7.8	7.8
4	19.2	12.0	12.0				

These tables bear out the conclusion that the time of passage is inversely proportional to the rate of revolution of the kiln.

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EFFECT OF LENGTH OF KILN

The effect of the length of the kiln is self-evident. The rate of advance per foot is what is being measured, so the time of passage is directly proportional to the length. The reader should note, however, that this is not absolutely true when a very short kiln is compared to a very long one. The piling-up effect mentioned under the study of the effect of the slope is an "end" phenomenon and is apt to cause disturbing effects in short kilns at low angles with heavy feed. Moreover, at the discharge end of the kiln there is a "dropping-off" effect, which is explained by the fact that the particles in coming near the open discharge have nothing to support them and instead of the uniform bed of material going entirely to the end and then falling off there is a gradual fall which reaches back several inches in the smaller kilns, and probably farther in commercial kilns of large diameter. Piling up and dropping off are both end effects, hence under certain conditions direct comparison of very short and very long kilns is not strictly possible.

EFFECT OF DIAMETER OF KILN [1]

As stated, kilns of four different sizes were available, a 3¾-inch kiln 5 feet long, and 6-inch, 11½-inch, and 19¾-inch kilns 7 feet long. In comparing the 3¾-inch to the other kilns the time of passage was multiplied by 7/5 to make the correction for difference in length. The following tables contain data which prove that the time of passage is inversely proportional to the diameter.

In Table 12 the calculated time in column 3 is found by multiplying the observed time in the 3¾-inch kiln by $\frac{3.75}{11.5} \times 7/5$.

TABLE 12.—*Effect of diameter on time of passage of Ottawa standard sand in 3¾-inch and 11½-inch kilns at 1 r. p. m.*

Angle, degrees	Time of passage, minutes		Angle, degrees	Time of passage, minutes	
	Observed	Calculated		Observed	Calculated
3	20.4	25.0	8	9.0	9.1
4	17.8	17.3	10	6.9	6.9
5	14.4	14.6	12	5.7	5.7
6	11.65	11.4			

Tables 13 and 14 give data on the 6-inch, 11½-inch, and 19¾-inch kilns.

[1] Some of the measurements in the 19¾-inch kiln were made by B. Reynolds, department of mining, University of California.

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TABLE 13.—*Comparison of time of passage of Ottawa standard sand in 19¾-inch and 11½-inch kilns at 1.6 r. p. m.*

Angle, degrees	Observed time in 19.75-inch kiln, minutes	Calculated time in 11.5-inch kiln, minutes	Angle, degrees	Observed time in 19.75-inch kiln, minutes	Calculated time in 11.5-inch kiln, minutes
3	8.8	9.1	5	4.9	4.9
4	6.1	6.3	6	4.0	4.1

TABLE 14.—*Comparison of time of passage of Ottawa standard sand in 6-inch and 19¾-inch kilns*

Angle, degrees	Time of passage observed in 6-inch kiln, minutes	Time of passage in 19¾-inch kiln, minutes		Angle, degrees	Time of passage observed in 6-inch kiln, minutes	Time of passage in 19¾-inch kiln, minutes	
		Observed	Calculated			Observed	Calculated
1	66.3	21.8	20.2	4	21.8	6.1	6.6
2	43.2	13.0	13.1	5	17.5	4.9	5.3
3	28.8	8.8	8.8	6	14.5	4.0	4.4

With the exception of the 6-inch tube, the shells were not entirely uniform in cross section, nor were the cross sections true circles. Therefore, the above results are quite satisfactory and serve to justify the conclusion that the time of passage of a particle through a rotary kiln varies inversely as the diameter of the kiln when other factors are constant.

ANGLE OF REPOSE

By watching the advance of particles through a rotary kiln and tracing their path on the shell of the kiln it is found that the particles are carried up the kiln in a mass of material which is quiet relative to the kiln; then, after reaching a position where the angle of repose of the material is exceeded, roll down to the bottom of the kiln again and in so doing advance toward the outlet because of the inclination of the axis of the kiln. The actual path of a particle on the shell of the kiln is a helix. The average particle follows a helix whose diameter is somewhat less than that of the kiln.

The writers observed that when the kiln was rotating very slowly the material stuck to the kiln, and after the angle of repose was exceeded it would suddenly rush down until the angle of the surface to the horizontal was such that sliding or rolling friction stopped the movement. If the rate of rotation of the kiln was more rapid, the "slumping" or sudden rushing was not observed and there was instead a continuous sheet of particles rolling down the surface of the mass in the kiln. These two forms of falling were named "slumping" and "continuous fall."

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DEFINITION OF ANGLE OF REPOSE

If the condition of slumping is analyzed further, it is seen that when the kiln is rotated slowly all the mass of material in it which occupies a sector of the cross section of the kiln sticks together until rotation of the kiln has carried it up to the angle where it starts to roll or slide down the steep slope so created. This is the angle which is designated as the angle of repose and is defined as the angle beyond which the material will not stand and will start to slide or roll. It will be recalled that starting friction is greater than sliding friction, and in accordance with this rule the material, after once starting to slide in a slowly moving kiln, will roll or slide rapidly down until the angle of the surface with the horizontal is such that sliding friction stops the motion and the whole mass of material comes to rest again. This final angle might be another angle of repose and, in fact, is the one usually described by geologists, engineers, and others who are interested in the angle at which material stands when it comes to rest. The authors, therefore, use the angle of repose to denote the condition where motion is just about to begin instead of where motion is just about to cease.

All the peculiarities of the material, such as smoothness, shape of particles, density of particles, and other physical factors, are reflected in the angle (or angles) of repose; therefore, the material is defined enough for present purposes when its angle of repose is measured. The effects of different factors on the angle of repose are also interesting and will therefore be discussed at some length.

EQUIPMENT FOR MEASUREMENT OF ANGLE OF REPOSE

A method of measuring the angle of repose was devised in the laboratory and is illustrated in Figure 6. A tin can about 17.3 cm. in diameter and 24.1 cm. long was mounted horizontally upon a circular wooden block, which in turn was set on a pillow block so that it was free to rotate. A smaller can 13.2 cm. in diameter and 21.6 cm. long was also used. The front end of the can was cut out and a plane plate of glass was clamped on it. A 360° circular protractor made of paper was mounted on the bench behind the can, with the axis of the protractor in the same line as the axis of the pillow block, which in turn was in line with the axis of the can. A horizontal bar was mounted on the floor in front of the can, and a bevel protractor on this bar was used to measure the angle of the material in the can. A pointer was put on the wooden block carrying the can and used for reading angles on the circular protractor mounted behind the can. A given quantity of the material to be measured (1,000 c. c. is convenient for a can of this size) is inserted, the glass front clamped on, and the can rotated. When the can is rotated very slowly to a certain point, the material in it slumps. The angle after the slump can be read with the bevel protractor, and

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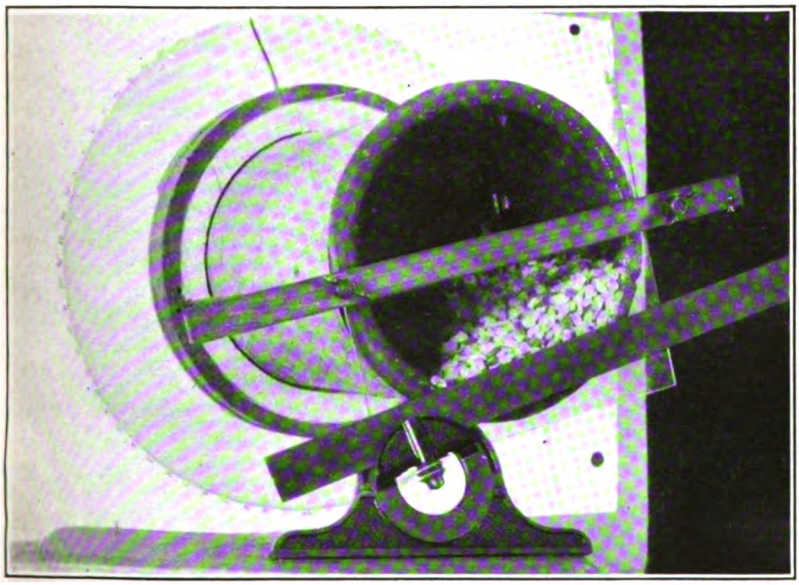

FIGURE 6.—Laboratory apparatus for measuring angle of repose

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the number of degrees necessary to rotate the can in order to get a slump can be read with the pointer and the circular protractor. Evidently, the angle after the slump plus the angle through which the can rotated before the slump is the angle of repose.

METHOD OF MEASUREMENT

In the Bureau of Mines laboratory the angle of repose was measured as follows: The can was rotated until the bed was uniform throughout the entire length; then the can was rotated just enough to cause a slump. The angle on the circular protractor was read. The can was then rotated until another slump took place, and the angle after slump was measured with the bevel protractor—so on for 10 measured slumps. Then the angle on the circular protractor (the angle traveled in the 10 slumps) was read. The average angle after slump plus the average peripheral change equals the angle of repose. Usually 5 sets of 10 slumps each were measured, so the angle of repose recorded was the average of 50 separate readings.

ADVANTAGES OF METHOD

This method of measuring the angle of repose is simple and rapid. If the material is in large chunks, a larger can could be used, or even half a barrel. Furthermore, to have the can or barrel mounted is not absolutely necessary. It might be rolled on a smooth floor, the distance traveled in a given number of slumps measured, and the angle calculated from this.

For ordinary angular particles the angle of repose has been found to be virtually independent of the lining of the can as long as its surface is rough, but if the surface is smooth the material in the can slips around too much. Table 15 illustrates this.

TABLE 15.—*Relation between lining of can and angle of repose*

Material	Lining of can [1]	Angle of repose, degrees
Ottawa standard sand	12 to 20 mesh quartz	35.89
	30 to 50 mesh quartz	35.22
	80 to 100 mesh quartz	35.08
	Ottawa standard sand	35.04
30 to 50 mesh quartz	12 to 20 mesh quartz	42.18
	30 to 50 mesh quartz	42.81
	80 to 100 mesh quartz	42.27
	Ottawa standard sand	42.33

[1] Held by glue.

EFFECT OF VARIABLES ON ANGLE OF CONTINUOUS FLOW

The particles sometimes slump, but if a certain critical rate of rotation of the kiln is exceeded they fall in continuous flow. To study the effect of speed of rotation and depth of bed on this occurrence, a set of cone pulleys was made and the speed of rotation at which continuous fall began with different depths of bed was studied.

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When the volume of the material in the can was plotted as the abscissa and the critical speed as the ordinate, the value of y decreased rapidly with increasing feed and then became nearly constant over a rather wide range. Table 16 and Figure 7 together give the data. When, however, the volume of the material in the can approached one-half of the total volume of the can, the critical speed rose slightly. With a large can and a depth of bed of only a few particles of material the critical speed first increased and then dropped off as usual. By using four different cans of the same length but of different diameters ($d = 7.9$ cm., 12.8 cm., 17.3 cm., and 21.5 cm.) the speed of rotation which just brought about continuous fall for the same volume of material varied as the square of the radius of the can, except for very thin or very deep beds of material. The work done does not justify

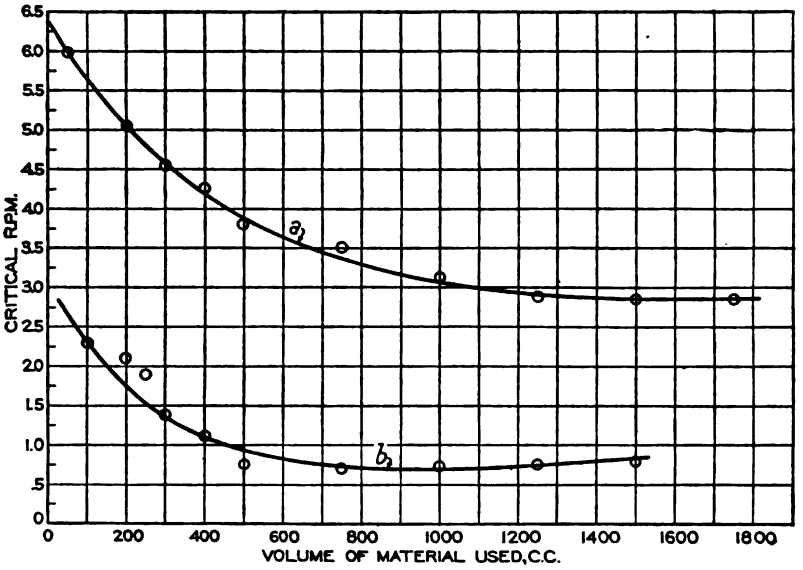

FIGURE 7.—Critical r. p. m. at which continuous flow occurs with different volumes of material; total volume of can, 5,660 c. c.: *a*, 20 to 30 mesh quartz; *b*, Ottawa standard sand

a more thorough discussion of the subject, but some one interested may study the factors that involve continuous flow and those that cause the abnormal results with very low and very high feeds.

TABLE 16.—*Minimum rate of rotation for continuous flow in small can* [1]

Material	Volume, c. c.	R. p. m.	Material	Volume, c. c.	R. p. m.
Ottawa standard sand	100	2.30	20 to 30 mesh quartz	100	5.98
	200	2.10		200	5.05
	250	1.90		300	4.54
	300	1.40		400	4.26
	400	1.14		500	3.81
	500	.76		750	3.52
	750	.70		1,000	3.13
	1,000	.73		1,250	2.90
	1,250	.75		1,500	2.87
	1,500	.80		1,750	2.87

[1] Can, 13.2 cm. diameter, 21.6 cm. long, 2,960 c. c. volume.

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The angle of continuous flow of the material in the can was studied. Results showed that the angle was constant regardless of the amount of material in the can (250 to 1,000 c. c.), that the angle remained constant throughout a wide range of speed of rotation, and that the angle was between the angle of repose and the angle after slip. For Ottawa standard sand the angle of repose was 34.4°, the angle of continuous flow 31.7°, and the angle after slip 29.2°. For 12 to 20 mesh crushed quartzite θ was 43.4°, the angle of continuous flow 37.7°, and the angle after slip 33.9°.

Knowledge of the speed at which continuous fall takes place may prove to be valuable in certain industrial plants because of its bearing on the elimination of loss by dusting. No other application of the phenomenon is obvious at present.

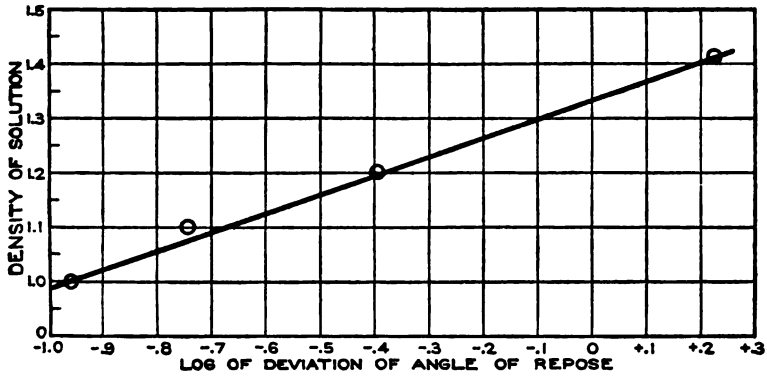

FIGURE 8.—Effect of density of medium on angle of repose of Ottawa standard sand

EFFECT OF LIQUID MEDIUM ON ANGLE OF REPOSE

The effect of a liquid medium on the angle of repose was studied. Measurement of the angle of repose of several materials in water at temperatures ranging from 20 to 75° C. showed that the angle differed very little from the angle in air and that within the limits of experimental error no difference attributable to the different temperatures of water could be detected. The angle of repose of Ottawa standard sand in air was 34.34°; in water at 20.5° C. it was 34.45°. The mean of about 25 readings between 25 and 75° C. was 34.44°.

A study was made of the effect of the density of the liquid medium on the angle of repose. Table 17 and Figure 8 show that the graph of the density of solution plotted against the log of the deviation of the angle of repose in air is a straight line. Interpretation of the physical meaning of this fact is difficult and, as the fact seems to have no practical value, has not been attempted.

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TABLE 17.—*Effect of density (d) of medium on angle of repose*

Medium	Angle of repose, degrees	Deviation of angle of repose, degrees	Log of deviation
Air	34.34		
H_2O ($d=1$)	34.45	0.11	−.9547
"Hypo" solution:			
$d=1.1$	34.53	.19	−.7212
$d=1.2$	34.74	.42	−.3979
$d=1.41$	35.97	1.63	+.2122

EFFECT OF SIZE OF PARTICLES ON ANGLE OF REPOSE

The angle of repose for particles smaller than 10 mesh was very nearly independent of the size of the particles. A sample of 12 to 20 mesh crushed quartzite had an angle of repose of 43.4°; 30 to 50 mesh, 42.5°; and 50 to 80 mesh, 42.6°. As the size of particles increases, however, the angle of repose increases. For 20 to 30 mesh quartz θ was 40.6° and for 3 to 4 mesh was 48.7°. A mixture of 1 part 20 to 30 mesh to 1 part 3 to 4 mesh had a θ of 42.7°; 2 parts 3 to 4 mesh to 1 part 20 to 30 mesh, 43.2°; 3 parts 3 to 4 mesh to 1 part 20 to 30 mesh, 45.1°.

For this one material, at least, the different sizes have different angles of repose, and the minimum angle is for material of 20 to 30 mesh size. Mixtures of different sizes have angles of repose intermediate between those of the separate sizes.

EFFECT OF ANGLE OF REPOSE ON TIME OF PASSAGE OF PARTICLES THROUGH ROTARY KILNS

It has been shown that:

$$t \propto 1/p$$
$$t \propto 1/n$$
$$t \propto 1/d$$
$$t \propto l$$
t = independent of rate of feed.

Introducing a proportionality constant k,

$$k = \frac{tpdn}{l}$$

This equation might itself be called the "kiln equation." The time of passage, however, varies with the angle of repose of the material. It is a function of the angle of repose of the material while the kiln is in motion, but measurements have shown that the angle of repose as measured in the small can is virtually identical with the highest angle the material reaches in the kiln while in motion. As this leads to simple measurements before the kiln is constructed, it has been chosen as the most satisfactory, useful, and convenient. The material rises to a certain point in the kiln and

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then falls. The height to which it rises is a function of the angle of repose. The time of falling is negligible compared to the time of rising; hence the angle of repose is the determining element of time of passage.

ANGLE OF REPOSE OF VARIOUS MATERIALS

The angle of repose of several materials was measured and k from 2 to 8° calculated (measurements made in the 6-inch kiln); it was found that plotting log k v. log θ produced a curve closely approaching a straight line. Table 18 and Figure 9 give the data.

TABLE 18.—*Table of angle of repose and k*

Material	θ	$\text{Log}_{10}\theta$	k	$\text{Log}_{10}k$
Lead shot	24.4	1.3874	7.19	0.8567
Millet seed	32.8	1.5159	9.00	.9542
Ottawa standard sand (20 to 30 mesh)	34.4	1.5366	9.96	.9983
River sand (20 to 50 mesh)	38.7	1.5877	10.64	1.0269
Quartz (20 to 30 mesh)	40.6	1.6085	10.92	1.0382
Coal (20 to 30 mesh)	41.7	1.6201	11.25	1.0512
Copper slag (20 to 30 mesh)	42.4	1.6274	10.96	1.0398
Stibnite (20 to 30 mesh)	42.5	1.6284	11.14	1.0469
Coke (20 to 30 mesh)	44.3	1.6464	11.85	1.0737
Beet seed	50.9	1.7067	11.49	1.0603
Sawdust	55.6	1.7451	12.40	1.0934

As the plot approaches a straight line the equation is of the form:

$$k = a\theta^n$$

or $$\log k = \log a + n \log \theta$$

Let $x = \log k$

$y = \log \theta$

When $x_2 = 1.733$; $y_2 = 1.10$

$x_1 = 1.530$; $y_1 = 1.00$

$$y = nx + b$$

$$n = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1.10 - 1.00}{1.733 - 1.53} = \frac{0.10}{0.203} = 0.492 = n$$

$$1.00 = (0.492)\ (1.53) + \log a$$

$$\log a = 0.2472,\ a = 1.77$$

$$k = 1.77\theta^{.492}$$

or approximately $$k = 1.77\sqrt{\theta}$$

which is approximately $$k = \sqrt{\pi\theta}$$

or $$t = \frac{1.77\, l\sqrt{\theta}}{pdn} = \frac{l\sqrt{\pi\theta}}{pdn} \text{ (approx.)}.$$

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Lead shot seem more abnormal than any of the other materials studied. During the first run with this material the abnormality was even greater ($k = 4.56$), but when the inside surface of the kiln was relined with coarser material the error was lowered from about 50 to about 15 per cent. As shot are round, they tend to roll down the kiln and thus travel through it too fast. Making the inside of the kiln rougher largely eliminated this tendency, which could probably be reduced even more by making the inside still rougher.

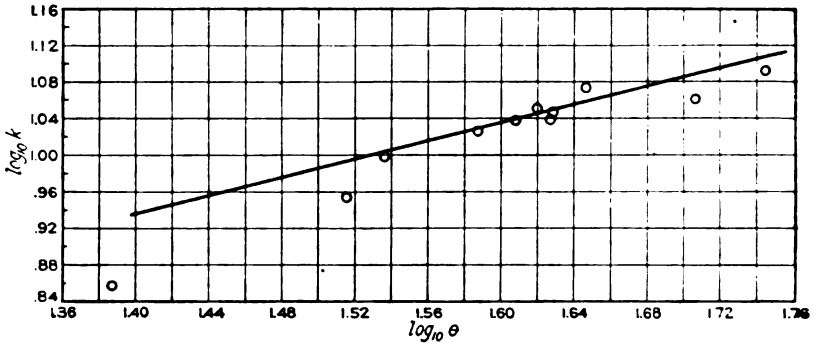

FIGURE 9.—Variation of kiln constant with material

EXAMPLE OF USE OF FORMULA

The following tables show how well the formula holds for some of the materials studied in the laboratory kiln:

TABLE 19.—*Calculated and observed time of passage of some materials through the 6 by 84 inch kiln*

Material	Angle, degrees	Calculated time, minutes	Observed time, minutes
River sand	2	30.8	30.1
	3	20.5	20.0
	4	15.4	14.8
	5	12.3	12.0
	6	10.2	9.9
	8	7.7	7.1
Coal (20 to 30 mesh)	2	32.0	32.7
	3	21.3	21.4
	4	16.0	16.2
	5	12.8	12.5
	6	10.6	10.3
	8	8.0	7.5
Stibnite (20 to 30 mesh)	2	32.2	32.5
	3	21.5	21.4
	4	16.1	15.7
	5	12.9	12.7
	6	10.7	10.3
	8	8.05	7.5

These typical data suffice to show how well the formula holds. The error will be greater for the two extremely unusual materials, lead shot and sawdust, as these points were farther from the straight

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line on the logarithmic curve. The very close approximation of the calculated to the observed time at 2, 3, and 4°, the working angles in commercial kilns, is to be noted. With the exception of such abnormal materials as lead shot and sawdust, the average mean error in the calculated time of passage of the materials studied was less than 6 per cent.

EFFECT OF TEMPERATURE IN LABORATORY KILN

Two runs were made with the 3¾-inch kiln at about 900° C. to test the effect of temperature. One run was made with Ottawa standard sand, a material that neither sinters nor gives off gas during heating. It was found that temperature had very little influence on time of passage, as Table 20 shows.

TABLE 20.—*Comparison of Ottawa standard sand in cold and in hot furnace*

Angle, degrees	Time of passage (cold), minutes	Time of passage (hot), minutes	Angle, degrees	Time of passage (cold), minutes	Time of passage (hot), minutes
5	7.57	7.3	10	3.7	3.5
6	6.15	6.1	12	2.97	2.94
7-9	4.7	4.5			

A run was made with magnesite, a material that gives off gas during heating. Here it was found that the time of passage was materially lessened when the temperature was raised. Table 21 gives the data at 6 and 8°.

TABLE 21.—*Comparison of magnesite in hot and in cold furnace*

Angle, degrees	Time of passage (cold), minutes	Time of passage (hot), minutes
6	7.95	7.55
8	5.42	4.92

In commercial kilns, such as cement kilns, where there is a period of drying, then calcining, and finally clinkering, the physical condition is likely to be different for each stage and correspondingly affects the time of passage.

EFFECT OF WETTING PARTICLES

Ottawa standard sand and quartz particles were wet with oil. Oil was used instead of water because the evaporation is very slight and the same degree of wetness would prevail for some time. The angle of repose changed greatly, but the time of passage was nearly the same in both the dry and the wet condition. Table 22 for Ottawa standard sand and Table 23 for quartz give the data.

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This failure of moist material to follow the kiln formula would be disturbing were it not for the observation that this material has virtually the same time of passage as when dry. It was also found that for coal wet with 7 per cent by weight of oil, the time of passage changed but little and was very nearly the same as in the dry condition, although the angle of repose changed from 41.7 to 60.8. It is necessary, then, to measure the angle of repose of dry material.

TABLE 22.—*Comparison of the time of passage of Ottawa standard sand when dry and when wet with 1 per cent by weight of oil*

Angle, degrees	Time of passage when dry (θ=34.4), minutes	Time of passage when wet (θ=46.3), minutes
2	28.2	27.8
4	14.2	13.7
6	9.1	9.1

TABLE 23.—*Comparison of the time of passage of quartz when dry and when wet with 1.5 per cent by weight of oil*

Angle, degrees	Time of passage when dry (θ=40.6), minutes	Time of passage when wet (θ=50.2), minutes
2	29.5	29.0
4	15.4	14.9
6	10.0	9.8

EFFECT OF CONSTRICTIONS OR "NOSE RINGS"

An investigation of the effect of constrictions in determining the time of passage of particles through the kiln seemed desirable.

FORMATION OF NOSE RINGS

"Nose rings" are often formed in kilns, usually at a point rather close to the place where sintering starts. In cement kilns these rings are formed about 20 to 25 feet from the discharge or flame end. The size and width of nose rings may vary, but ordinarily they are not allowed to become too large. Constrictions are also sometimes built in kilns to increase the time of passage.

POSSIBLE CONDITIONS OF BED

There are three possible conditions of shape of bed relative to the kiln and nose ring. The first is condition *c* in Figure 10, where the feed produces such a depth of bed that if a constriction was absent the average depth of bed of the material would be less than the height

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of the nose ring. The depth of bed which would be produced by a given rate of feed in an unobstructed kiln is called the "uniform bed." In case *c* a condition of uniform bed for a part of the kiln results; then follows a certain piling up near the nose ring, which permits the passage of material through the constriction. Case *b* (fig. 10) illustrates a condition where the uniform depth of the bed is equal to the height of the constriction. Case *a* shows a condition where the uniform depth of the bed is greater than the height of the constriction.

ADDITION OF NOSE RINGS TO KILN

To study the effect of constrictions in the laboratory kiln, observations were made with a nose ring in different positions in the kiln, with

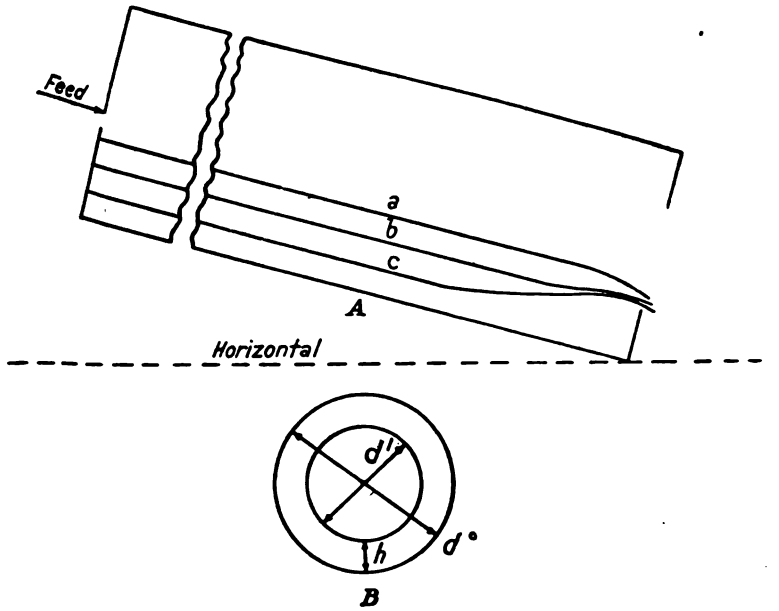

FIGURE 10.—*A*, Longitudinal section of kiln, showing different conditions of bed of material when nose ring is present; *B*, end section with nose ring present

different slopes of the kiln, with different speeds of rotation, and with different rates of feed. These measurements were made in the 7-foot by 6-inch kiln. All measurements were made with Ottawa standard sand as the solid material passing through the kiln; usually the kiln was rotating at 2.5 r. p. m.

EFFECT OF 5½-INCH WOODEN NOSE RING

A wooden nose ring 5½ inches long and with an opening 4½ inches in diameter (the constriction was thus 0.75 inch high) was used first. Observations were made with different rates of feed and with the constriction placed at the discharge end of the kiln, 1 foot from the

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discharge end, and 2 feet from the discharge end. Table 24 and Figure 11 give the data.

TABLE 24.—*Wooden diaphragm, 5.5 inches long; diameter of hole, 4.5 inches*

Distance from discharge end of kiln, feet	Feed per 15 minutes, c. c.	Time of passage, minutes	Distance from discharge end of kiln, feet	Feed per 15 minutes, c. c.	Time of passage, minutes	Distance from discharge end of kiln, feet	Feed per 15 minutes, c. c.	Time of passage, minutes
0	433	30.7	1	437	30.2	2	437	31.0
	875	27.3		890	26.5		995	25.9
	3,100	25.3		1,550	24.2		1,620	24.0
	4,180	27.0		3,190	23.5		3,160	23.0
	5,004	28.2		5,510	23.4			
	5,835	29.2						

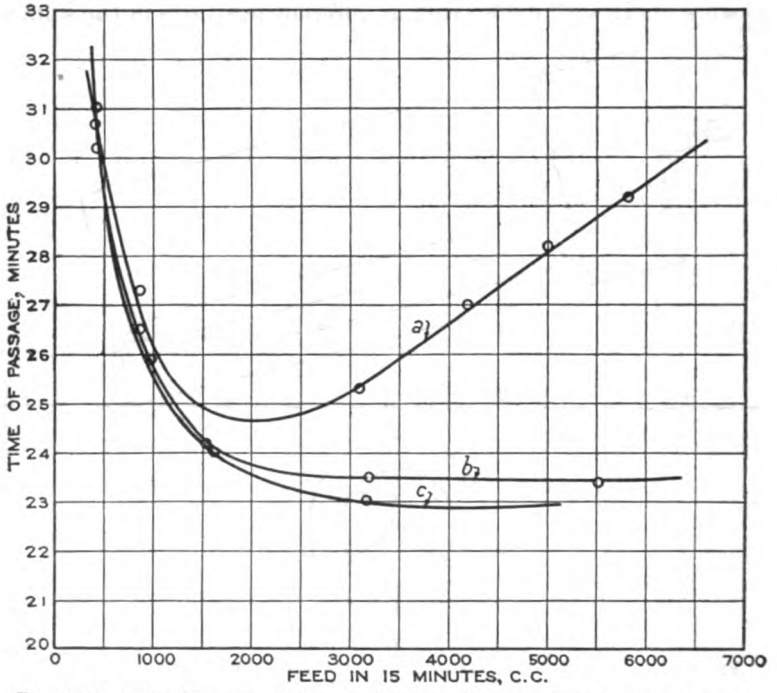

FIGURE 11.—Effect of wooden diaphragm 5.5 inches long with 4.5-inch diameter opening; diameter of kiln, 6 inches: *a*, Diaphragm at discharge end of kiln; *b*, diaphragm 1 foot from discharge end of kiln; *c*, diaphragm 2 feet from discharge end of kiln

The curve always drops off rapidly at first as the feed increases, but, except when the constriction is at the end, at about 1,900 c. c. per 15 minutes it goes off at a straight line; that is, the time of passage approaches a constant. Observations showed that about this feed gave uniform depth of bed, equal to the height of the constriction. When the nose ring was at the discharge end of the kiln, the curve passed through a minimum as the feed increased. Explanation of this may be that when there is no constriction at the discharge end there is a gradual "dropping-off" effect, but when there is a constriction this effect is impossible. Thus, a greater feed

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causes a greater piling up, which in turn increases the time of passage. When the nose ring is placed back from the discharge end, the usual diaphragm-free end effect is again present. It will be noted that when the feed is 400 to 1,900 c. c. per 15 minutes the time of passage is virtually the same regardless of the position of the diaphragm, and that at 1 and 2 feet from the discharge end the time is virtually identical throughout the entire range. Artificial nose rings would not ordinarily form at the discharge end in commercial kilns.

EFFECT OF FIBER NOSE RING

To check up on the curve passing through a minimum when the constriction was at the discharge end of the kiln, a fiber diaphragm of the same diameter opening, but only about three-sixteenths inch in width, was obtained and a series of measurements made. Table

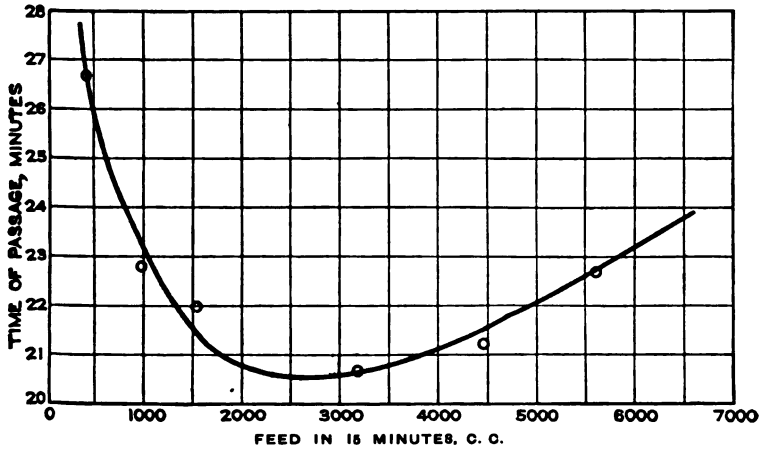

FIGURE 12.—Effect of fiber diaphragm with 4.5-inch diameter opening at discharge end of 6-inch kiln

25 and the accompanying graph, Figure 12, show that the same general shape of curve is obtained, although the time of passage was much less than when the wider diaphragm was used.

TABLE 25.—*Results with 4.5-inch diameter hole, fiber diaphragm, at discharge end of kiln*

Feed per 15 minutes, c. c.	Time of passage, minutes	Feed per 15 minutes, c. c.	Time of passage, minutes
425	26.7	3,180	20.7
796	22.8	4,460	21.2
1,550	22.0	5,625	22.8

EFFECT OF POSITION OF NOSE RING

A study was made of the effect of the position of the diaphragm in the kiln with a given diaphragm at a definite rate of speed. Three

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diaphragms were made of three-sixteenths-inch fiber; one had a 4.5-inch diameter opening, one a 3.75-inch, and the other a 3-inch opening. The accompanying Table 26 and the corresponding Figure 13 show that when the feed is constant the position of the diaphragm in the kiln has practically no effect on the time of passage. This statement does not apply strictly to the 3-inch diameter hole at a very low feed, as the time decreases a little as the diaphragm is moved back from the discharge end; but it soon reaches a constant value.

TABLE 26.—*Effect of position of the diaphragm in the kiln at a given rate of feed n=2.5*

Diameter of diaphragm hole, inches	Distance from discharge end of kiln, inches	Feed per 15 minutes (approximate) c. c.	Time of passage, minutes
4.5	0	3,175	20.7
	3		20.1
	6		19.7
	9		19.6
	12		19.6
	15	2,280	19.7
	18		19.4
	24		19.3
	30		19.1
	36		19.2
3.75	0	410	26.7
	3		26.7
	6		27.5
	9		26.6
	0	2,280	22.7
	12		21.7
	24		21.4
	36		21.3
3.0	0	410	59.6
	3		54.7
	6		53.0
	9		53.5

The constrictions put in the kiln were relatively large in comparison with the kiln itself. For the diaphragm with the 4.5-inch hole the cross-sectional area of the opening is only 56.5 per cent of that of the kiln itself, and the area of the 3-inch hole is only 25 per cent. In commercial kilns the nose rings would not be allowed to grow this large.

RESULTS

The results given in the foregoing tables bring out these facts:

1. If the uniform depth of bed is as great as the height of the constriction (the constriction being thin or narrow), the time of passage is virtually the same with the diaphragm in different parts of the kiln.

2. For a given position of the constriction, after the uniform depth of bed reaches the height of the constriction, the time of passage is constant with increasing feed, unless the constriction is at the discharge end of the kiln, and then the time of passage increases with increasing feed.

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3. When the uniform depth of bed is less than the height of the constriction, the time of passage is greater than when no constriction is present but is independent of the position of the constriction.

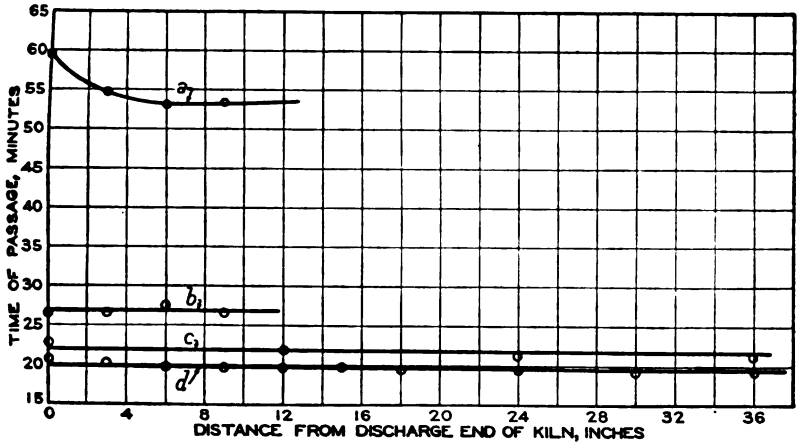

FIGURE 13.—Effect of position of diaphragm in 6-inch kiln: *a*, Fiber diaphragm, 3-inch diameter opening, feed about 410 c. c. per 15 minutes; *b*, fiber diaphragm, 4.5-inch diameter opening, feed about 410 c. c. per 15 minutes; *c*, fiber diaphragm, 3.75-inch diameter opening, feed about 2,280 c. c. per 15 minutes; *d*, fiber diaphragm, 4.5-inch diameter opening, feed 2,280 to 3,175 c. c. per 15 minutes

Comparison of Tables 24 and 25 and Figures 11 and 12 shows that the time of passage increases when the width of the diaphragm is increased.

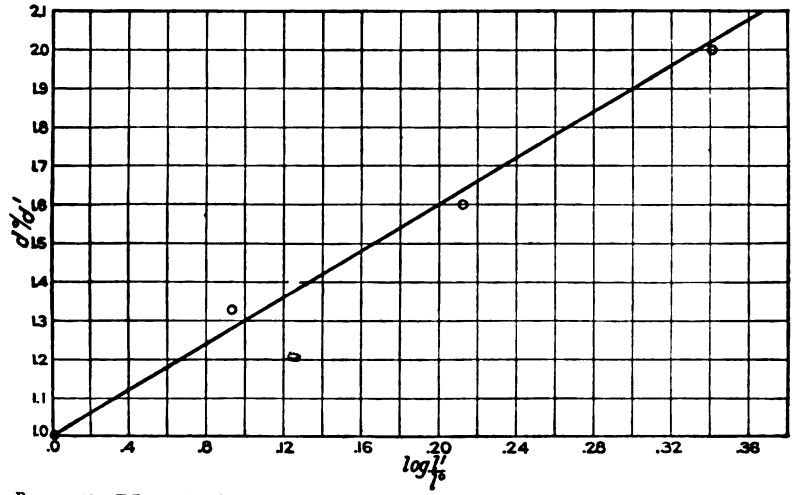

FIGURE 14.—Effect of ratio of diameter of kiln to diameter of nose-ring opening on change in apparent length of kiln; diameter of kiln, 6 inches; slope of kiln, 3°; speed of kiln, 2.5 r. p. m.

EQUATIONS FOR DIFFERENT CONDITIONS

The final step is to work out the equations for these conditions: (1) Where the uniform depth of bed is smaller than the height of the constriction, (2) where the uniform depth of bed is greater than the

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height of the constriction and the latter is at the discharge end of the kiln, and (3) where the uniform depth of bed is greater than the height of the constriction and the constriction is at a position other than the discharge end.

At a given pitch of the kiln, at 2.5 r. p. m., a constant feed was maintained and measurements made in the 6-inch kiln with no constriction, and with the 4.5-inch, the 3.75-inch, and the 3.0-inch thin fiber nose rings. The feed was always so small that the uniform depth of bed was not greater than 0.75 inch. The data are given in Table 27.

TABLE 27.—*Effect of constriction on time of passage when the uniform depth of bed of material is less than the height of the constriction; constriction, 3 inches from the discharge end of kiln; n=2.5*

Slope, degrees	Diameter of hole in diaphragm, inches	Feed per 15 minutes (approximate), c. c.	Time of passage, minutes
3	(1)	410	18.7
	4.5		26.7
	3.75		36.9
	3.0		54.7
	(1)	760	18.0
	4.5		22.3
	3.75		29.4
	3.0		39.4
	(1)	1,200	18.25
	4.5		21.6
	3.75		24.8
	3.0		31.9
	(1)	1,525	18.3
	4.5		20.8
	3.75		23.9
	3.0		29.0
4	(1)	1,150	13.9
	4.5		16.7
	3.75		19.5
	3.0		24.3
	(1)	1,500	14.0
	4.5		15.8
	3.75		18.1
	3.0		22.3
2	(1)	1,450	26.8
	4.5		30.0
	3.75		35.2
	3.0		42.7

[1] No constriction.

WORKING OUT THE EQUATIONS

Let d° = diameter of the kiln.

d' = diameter of the hole of the constriction.

l° = length of the kiln.

l' = apparent length of the kiln calculated on the basis of the time of passage when a diaphragm is present; that is,

$$l' = l^\circ \times \frac{\text{time of passage for } d'}{\text{time of passage for } d^\circ}$$

By plotting d°/d' against log l'/l° a curve closely approximating a straight line was produced. From this the equation involving the time of passage for a given feed with different constrictions can be obtained. An example will suffice to show this. The data are given in Table 28 and Figure 14.

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TABLE 28.—*Time of passage for given feed at different constrictions*

[Feed=approximately 760 c. c. per 15 minutes; angle=3°]

d'	$d°$	t	$l°$	l'	$d°/d'$	$l'/l°$	Log $l'/l°$
6.00	6.0	18.0	7.0	7.0	1.0	1.0	0.0000
4.5	6.0	22.3	7.0	8.67	1.33	1.24	.0934
3.75	6.0	29.4	7.0	11.4	1.60	1.63	.2122
3.0	6.0	39.4	7.0	15.3	2.0	2.19	.3404

The equation [2] of the curve is derived from Figure 14.

$$y = d°/d'$$
$$x = \log l'/l°$$

when $y_2 = 2.0;\ x_2 = 0.3345$

$$y_1 = 1.0;\ x_1 = 0.0000$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.0 - 1.0}{0.3345 - 0.0} = 2.99$$
$$y = mx + b$$
$$(d°/d') - 1 = 2.99 \log l'/l°$$
$$\frac{d° - d'}{d'} = 2.99 \log l'/l°$$
$$\log l'/l° = \frac{2h}{2.99d'} = ln\ l'/l° = \frac{4.6h}{2.99d'}$$
$$l' = l° e^{\frac{1.54h}{d'}};\ l' = l° e^{\frac{1.54h}{d° - 2h}}$$

where h = the height of the constriction; h and $d°$ must be expressed in the same units.

DETERMINATION OF K

For a given slope of the kiln this equation may be written

$$l' = l° e^{\frac{kh}{d° - 2h}}$$

where k is a constant depending on the feed. Similar calculations from the data in Table 27 at a slope of 3° give the following values of k. At a feed of 410 c. c. per 15 minutes, $k = 2.19$; at 1,200 c. c., $k = 1.06$; and at 1,525 c. c., $k = 0.88$. The value of k at a slope of 2° with a feed of approximately 1,450 c. c. per 15 minutes was 0.87; at 4° with a feed of 1,500 c. c., 0.90; and for 1,150 c. c., 1.10. These values are so close to those of 3° that for practical purposes at least the constant can be said to be independent of the slope of the kiln over the range of slopes studied, which includes those usually found in commercial kilns.

From the four determinations of k at 3° its value may be determined. The authors found that this value could be expressed in terms of volumes of feed relative to the volume of the kiln.

[2] In these equations e is the base of natural logarithms; ln signifies natural logarithm.

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The total volume of the kiln v_o is 39,100 c. c. Let v_f = the volume of feed per hour. By plotting k against $\log v_f/v_o$ a straight-line curve was produced. Table 29 and Figure 15 give the data.

TABLE 29.—*Effect of volume of feed on k* [1]

k	v_o	v_f	v_f/v_o	Log v_f/v_o
2.19	39,100	1,650	0.0422	−1.3747
1.54	39,100	3,050	.078	−1.1079
1.06	39,100	4,822	.123	−.9101
.88	39,100	6,100	.156	−.8069

[1] In these calculations v_f is the mean feed from which k was calculated.

$$y = k$$
$$x = \log v_f/v_o$$

from the curve,
when

$$y_2 = 2.0,\ x_2 = -1.32$$
$$y_1 = 1.0,\ x_1 = -0.86$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.0 - 1.0}{-1.32 - (-0.86)} = 1/-0.46 = -2.18$$
$$y = mx + b$$
$$2.0 = 2.88 + b$$
$$b = -0.88$$

hence

$$y = k = -2.18 \log v_f/v_o - 0.88$$

The equation $l' = l^\circ e^{\frac{kh}{d^\circ - 2h}}$ then reads

$$l' = l^\circ e^{(-2.18 \log v_f/v_o - 0.88)\frac{h}{d^\circ - 2h}}$$

CHECK OF CALCULATIONS

Table 30 contains some calculations to show how closely the exponential part of the equation holds as checked in the laboratory kiln. The calculated time was obtained by multiplying the time of passage with no constriction by the value of the exponential factor.

TABLE 30.—*Check of calculations with laboratory results*

Slope p, degrees	Feed per 15 minutes, c. c.	v_o	v_f	v_f/v_o	Log v_f/v_o	d'	h	Time of passage, minutes	
								Observed	Calculated
	1,150	39,100						18.0	
	1,264	39,100	3,180	0.0815	−1.0888	4.5	0.75	22.3	23.1
	1,214	39,100	3,090	.079	−1.1024	3.75	1.12	29.4	28.2
3	1,139	39,100	2,890	.074	−1.1308	3.0	1.5	39.4	39.8
	1,898							18.25	
	1,875	39,100	4,760	.122	−.9136	4.5	.75	21.6	21.6
	1,985	39,100	5,005	.129	−.8894	3.75	1.12	24.8	24.8
	1,904	39,100	4,825	.123	−.9101	3.0	1.5	31.9	31.7
	2,329	39,100						14.0	
4	2,402	39,100	6,100	.156	−.8069	4.5	.75	15.8	16.2
	2,341	39,100	5,950	.152	−.8182	3.75	1.12	18.1	18.3
	2,325	39,100	5,900	.151	−.8210	3.0	1.5	22.3	22.2
	2,282							26.8	
2	2,364	39,100	6,000	.153	−.8153	4.5	.75	30.0	31.1
	2,323	39,100	5,910	.151	−.8210	3.75	1.12	35.2	35.4
	2,357	39,100	5,980	.153	−.8153	3.0	1.5	42.7	42.4

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ADAPTATION OF FORMULA TO VARYING RATES OF ROTATION

These measurements were made at a speed of 2.5 r. p. m. For a given nose ring in a given kiln the piling-up effect is obviously a function of the depth of bed, which in turn is a function of the feed and the speed of rotation. If the kiln moves at a speed of x revolutions per minute and a feed y, there will be a certain piling-up effect. If the speed of rotation is changed to $2x$, the feed must be changed to $2y$ in order to produce the same depth of bed with the corresponding piling-up effect. To check this in the 6-inch kiln, a run was made

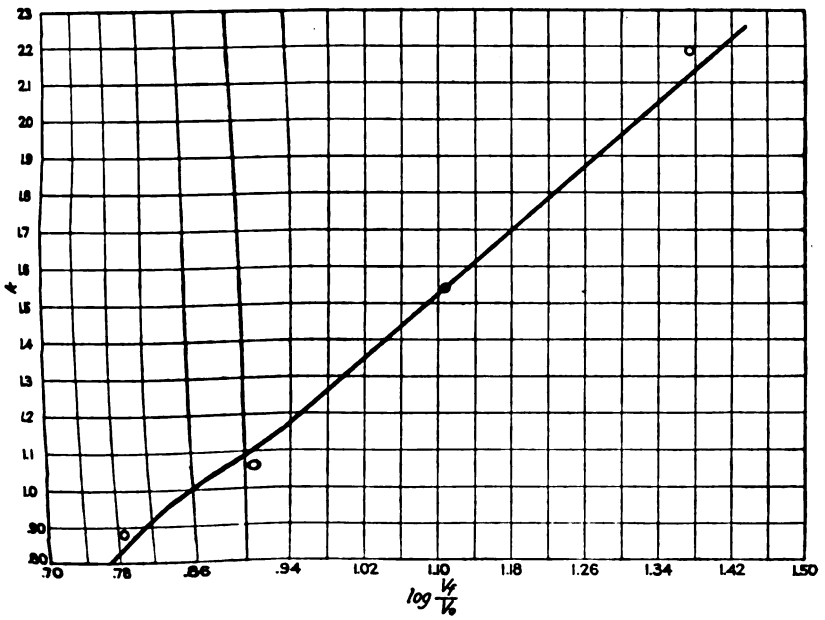

FIGURE 15.—Effect of rate of feed on k; diameter of kiln, 6 inches; slope of kiln, 3°; speed of kiln, 2.5 r. p. m.

at a speed of 1.25 r. p. m. As Table 31 shows, the foregoing assumption proved to be correct.

TABLE 31.—*Data on observations made at 1.25 r. p. m.*

Nose-ring opening, inches	Time of passage at 2.5 r. p. m., minutes	Time of passage at 1.25 r. p. m., minutes	
		Calculated	Observed
4.5	21.3	42.6	41.7
3.75	23.9	47.8	47.2
3.0	29.0	58.0	57.1

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The correct formula to account for revolutions per minute must now be written:

$$l' = l^{\circ}\, e^{\left(-2.18 \log \frac{2.5 v_f}{n v_o} - 0.88\right) \frac{h}{d^{\circ} - 2h}}$$

CORRECTION FOR KILNS OF DIFFERENT PROPORTIONS

This formula, however, holds only for this particular kiln or one of similar dimensions. The ratio of diameter to length is 6 : 84 or 1 : 14. From this ratio, however, the equation may be made to fit any kiln. For example, suppose a kiln to be 100 feet in length and 6 feet in diameter. The correction factor can be made to act for a kiln 6 feet in diameter by 84 feet in length and the remaining 16 feet of the kiln calculated on the basis of a simple unobstructed kiln from the simple formula,

$$t = \frac{1.77\, l\sqrt{\theta}}{pdn}$$

To check this in the laboratory kiln, a new kiln 3 inches in diameter by 7 feet long was set up. After this kiln was lined with sand in the usual manner some measurements were made in the same way as in the 6-inch kiln. The derived formula is based on the ratio of diameter to length of kiln of 1 to 14. As the new kiln was 3 inches in diameter, this same ratio would correspond to a kiln 42 inches long. The kiln actually was 84 inches long, hence 42 inches of it must be considered a straight unobstructed kiln and 42 inches a constricted kiln. The time of passage is calculated by adding the calculated time of passage for a 42-inch constricted kiln plus the calculated time of passage for a 42-inch simple kiln. In three observations with different-size nose rings at 2.5 r. p. m. and a 3° pitch of the kiln, the following are the calculated and the observed times, respectively: 46.5 minutes, 46.3 minutes; 48.0 minutes, 46.0 minutes; and 53.3 minutes, 52.1 minutes. These data tend to show that the assumption was justified.

CORRECTION WHEN RATIO IS LESS THAN 1 :14

The ratio of diameter to length may be less than 1:14. Actual measurement of the 6-inch kiln showed that the piling up (at least that noticeable or measurable) at 3° slope of the kiln never extended farther back into the kiln than a ratio of the diameter to the length of the piling up of 1:5 with the 3-inch diameter nose ring, of 1:4 with the 3.75-inch, and of about 1: 3.25 with the 4.5-inch diameter nose ring. With the aid of this information the exponential correction (that is, ratio of diameter to length) can be recalculated for any length of the kiln if the length is not less than the distance that the piling up extends back into the kiln.

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An example will be given to show how this may be done. Suppose that it is desired to express the correction in an exponential equation when the ratio of diameter to length is 1:7 instead of 1:14. It is known that the piling up does not extend back as far as a ratio of 1:7, hence the data of Table 27 may be used, with the assumption that the first 3.5 feet of the kiln act as a simple kiln without constrictions, and the last 3.5 feet act as the constricted part. One-half the time required for the material to pass through the unobstructed kiln is then subtracted from all the readings and from these data the corrected formula is derived. Table 32 gives the data.

TABLE 32.—*Data for calculating the correction factor for nose ring for ratio of diameter to length of 1. 7; n=2.5; slope, 3°*

Nose-ring opening, inches	Feed per 15 minutes, c. c.	t	$t/2$ [1]	$l°$	t' [2]	l' [3]	$l'/l°$	Log $l'/l°$
No constriction	410	18.7	9.35	3.5			1.0	0.0000
4.5	410	26.7	9.35	3.5	17.35	6.5	1.86	.2695
3.75	410	36.9	9.35	3.5	27.55	10.3	2.94	.4683
3.0	410	54.7	9.35	3.5	45.35	16.95	4.85	.6857
No constriction	760	18.0	9.0	3.5			1.0	.0000
4.5	760	22.3	9.0	3.5	13.3	5.17	1.48	.1703
3.75	760	29.4	9.0	3.5	20.4	7.93	2.27	.3560
3.0	760	39.4	9.0	3.5	30.4	11.8	3.37	.5276
No constriction	1,200	18.25	9.13	3.5			1.0	.0000
4.5	1,200	21.6	9.13	3.5	12.47	4.78	1.37	.1367
3.75	1,200	24.8	9.13	3.5	15.67	6.0	1.715	.2342
3.0	1,200	31.9	9.13	3.5	22.77	8.7	2.5	.3929
No constriction	1,525	18.3	9.15	3.5			1.0	.0000
4.5	1,525	20.8	9.15	3.5	11.65	4.45	1.27	.1038
3.75	1,525	23.9	9.15	3.5	14.75	5.64	1.615	.2081
3.0	1,525	29.0	9.15	3.5	19.85	7.59	2.17	.3365

[1] $t/2$=time of passage through first 3.5 feet, the unobstructed part of the kiln.
[2] t'=time of passage in constricted part (last 3.5 feet).
[3] l'=apparent length of constricted part.

Constant k.—Plotting $d°/d'$ v. log $l'/l°$ and obtaining the equations for the curves, it is found that k in the equation $l'=l° \; e^{\frac{kh}{d°-2h}}$ is 3.32 when v_f is 1,650; 2.46 for 3,050; 1.78 for 4,822; and 1.54 for 6,100. By plotting k v. log v_f/v_o it is found that the value of $k=-3.08 \log v_f/v_o-0.01$, hence substituting this value of k in the above equation, the general equation when the ratio of diameter to length is 1 : 7 is $l'=l° \; e^{(-3.08 \log 2.5 \, v_f/nv_o-0.01)\frac{h}{d°-2h}}$. Calculations made by the use of this formula showed that it holds satisfactorily.

Constants a and b.—The only change in the formula from that of the ratio of diameter to length of 1 : 14 is that of the two constants in this last equation. Substituting the general constant a for -3.08 and b for -0.01, a more general equation may then be written:

$$l'=l° \; e^{(a \log 2.5 \, v_f/nv_o+b)\frac{h}{d°-2h}}.$$

The constants a and b were calculated for different ratios of diameter to length and are recorded in Table 33.

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TABLE 33.—*Evaluation of constants a and b with different ratios of diameter to length*

Ratio of diameter to length	*a*	*b*	Ratio of diameter to length	*a*	*b*
1 : 5	−3.26	+1.0	1 : 12	−2.43	−0.77
1 : 7	−3.08	−.01	1 : 14	−2.18	−.88
1 : 10	−2.77	−.665			

On plotting a as abscissa against l/d as ordinate, a straight-line curve (fig. 16) results.

From Figure 16 the equation of the line is obtained, when

$$y_2 = 15;\ x_2 = -2.06$$

$$y_1 = 3;\ x_1 = -3.5$$

$$m = \frac{15-3}{-2.06-(-3.5)} = \frac{12}{1.44} = 8.33$$

$$y = mx + b$$

$$15 = (8.33)\ (-2.06) + b$$

$$15 = -17.2 + b$$

$$b = 32.2$$

$$y = l/d;\ x = a$$

$$y = mx + b$$

$$l/d = 8.33a + 32.2$$

$$8.33\ a = l/d - 32.2$$

$$a = \frac{l}{8.33d} - 3.86$$

$$a = 0.12\ l/d - 3.86$$

By plotting b as abscissa against l/d as ordinate a curve was produced which had the appearance of an exponential curve. As negative numbers were present, however, the logarithms could not be taken without the use of complex quantities. It was necessary, therefore, to transform coordinates so as to make all numbers positive. The curve (fig. 17) appears to be assymptotic to $b = 1$. Coordinates may therefore be transformed by this amount.

$$b = f(l/d) - 1$$

or

$$f(l/d) = b + 1.$$

Table 34 and Figure 18 contain the data.

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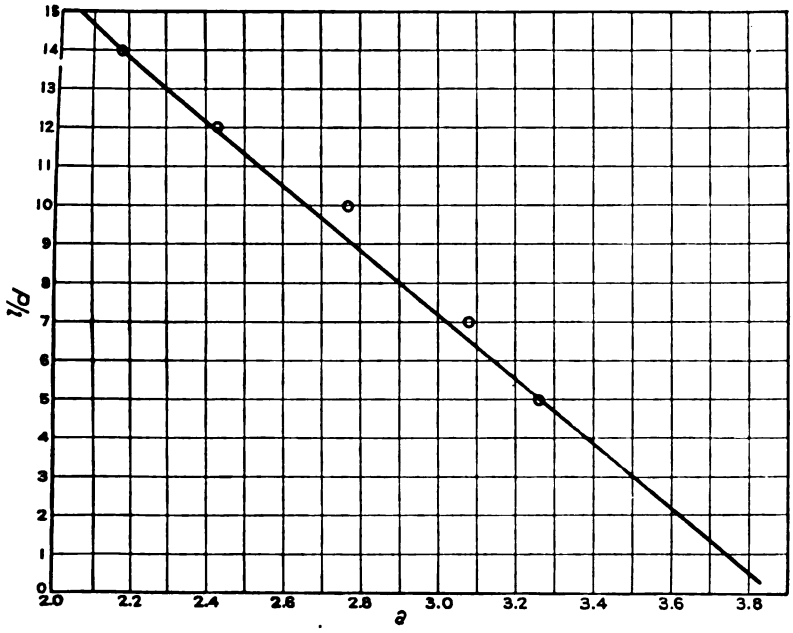

FIGURE 16.—Effect of ratio of length to diameter of kiln on constant *a* in general equation for the condition when the uniform depth of bed is less than the height of the nose ring

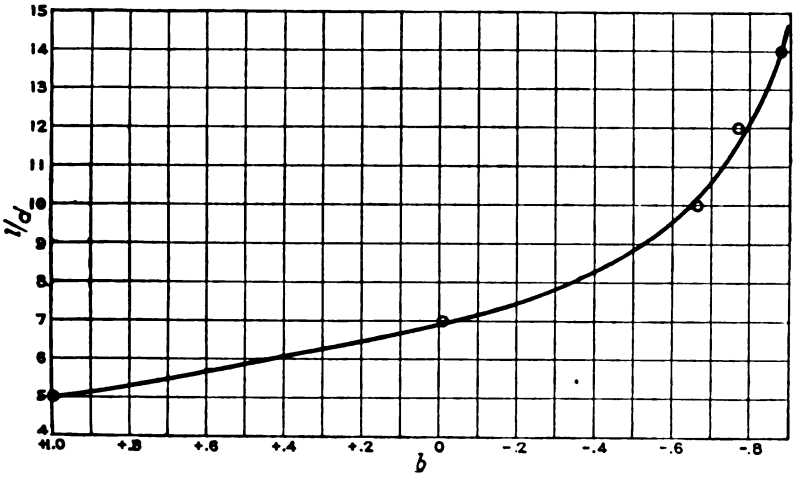

FIGURE 17.—Effect of ratio of length to diameter of the kiln on the constant *b* in the general equation for the condition when the uniform depth of bed is less than the height of the nose ring

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TABLE 34.—*Values of $f(l/d)$ obtained by transforming coordinates*

l/d	b	$f(l/d)$	Log $f(l/d)$	l/d	b	$f(l/d)$	Log $f(l/d)$
5	+1.0	2.0	+0.3010	12	−0.77	0.23	−0.6368
7	−.01	.99	−.0044	14	−.88	.12	−.9208
10	−.665	.335	−.4750				

$$\log f(l/d) = x$$
$$l/d = y$$

from the curve, when

$$y_2 = 15;\ x_2 = -1.09$$
$$y_1 = 5;\ x_1 = +0.301$$

$$m = \frac{15-5}{-1.09-0.301} = \frac{10}{-1.391} = -7.19$$

$$y = mx + b$$
$$15 = (-7.19)(-1.09) + b$$
$$15 = 7.83 + b$$
$$b = 7.17$$
$$l/d = -7.19 \log f(l/d) + 7.17$$

$$\log f(l/d) = \frac{7.17 - l/d}{7.19} = 1 - l/7.19d = 1 - 0.139l/d.$$

$$\ln f(l/d) = 2.3 - 0.32 l/d$$
$$f(l/d) = e^{2.3-0.32\,l/d}$$

then

$$b = e^{2.3-0.32\,l/d} - 1$$

The general equation (p. 31) then is:

$$l' = l^{\circ} e^{[(0.12l/d - 3.86)\log 2.5\, v_f/nr_0 + e^{2.3-0.321/d} - 1]\frac{h}{d^{\circ}-2h}}.$$

COMPLETION OF EQUATION

This exponential equation is the "factor" referred to in the equation given at the very first of this paper. The complete equation then is:

$$t = \frac{1.77\, l\sqrt{\theta}}{pdn} \times e^{\left[(0.12l/d - 3.86)\log \frac{2.5\, v_f}{nr_0} + e^{2.3-0.321/d} - 1\right]\frac{h}{d^{\circ}-2h}}.$$

This factor is 1 when no constriction or nose ring is present. The formula is for a kiln in which the feed is such as to give a uniform depth of bed less than the height of the nose ring.

Exponential functions may be evaluated as follows:

$$e = 2.7183;\ y = e^x$$
$$\log y = x \log e$$
$$\log y = 0.4343x$$
$$y = \text{antilog } 0.4343x$$

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ALTERNATE METHOD OF CALCULATING TIME OF PASSAGE

Still another method may be used to calculate the time of passage when nose rings are present, if the ratio of diameter to length of kiln is less than 1:14. A given kiln is 6 feet in diameter by 60 feet in length and it is desired to calculate the time of passage for a given material, with a given constriction, using the formula derived when $d:l::1:14$. The ratio of 6 to 60 is 1 to 10; hence an hypothetical 24 feet must be added to the kiln, which thus gives a ratio of 1:14, for purposes of calculation. The time of passage is then calculated from the exponential formula for an 84-foot kiln, and from this is

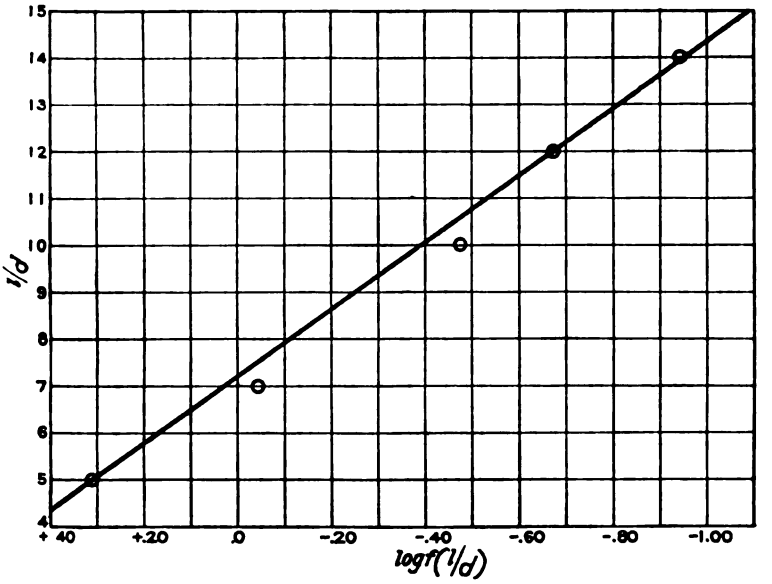

FIGURE 18.—Plot of l/d against $\log f(l/d)$, after transformation of coordinates

subtracted the time it would take the material to pass through a 24-foot unobstructed kiln, which gives the time of passage in the 60-foot kiln.

UNIFORM DEPTH OF BED OF MATERIAL GREATER THAN HEIGHT OF CONSTRICTION—CONSTRICTION AT DISCHARGE END OF KILN

As has been said, when the nose ring is at the discharge end of the kiln and the feed is sufficient to give a uniform depth of bed of material greater than the height of the nose ring, the time of passage is increased with increasing feed. Table 35 gives the data for nose rings with 4.0, 4.5, and 5.0 inch diameter openings, in the 6-inch laboratory kiln at a pitch of 3° and a speed of 2.5 r. p. m.

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TABLE 35.—*Data for uniform depth of bed of material greater than the height of constriction*

Diameter of nose-ring opening, inches	Feed per 15 minutes, c. c.	Time of passage, minutes	Diameter of nose-ring opening, inches	Feed per 15 minutes, c. c.	Time of passage, minutes	Diameter of nose-ring opening, inches	Feed per 15 minutes, c. c.	Time of passage, minutes
4.0	789	27.2	4.5	425	26.7	5.0	413	22.8
	2,222	22.4		797	22.8		449	20.5
	3,180	21.7		1,550	22.0		901	20.2
	4,435	21.8		3,181	20.7		1,351	18.3
	5,295	23.3		4,460	21.2		3,182	19.4
				5,615	22.8		5,665	21.2

PLOTTING OF CURVES

The time was plotted against the volume of feed per 15 minutes, and the time of passage for feeds intermediate between the observed points was taken from the smooth curve. The time of passage with no constriction was 19.5 minutes with feeds from 320 to 4,550 c. c.

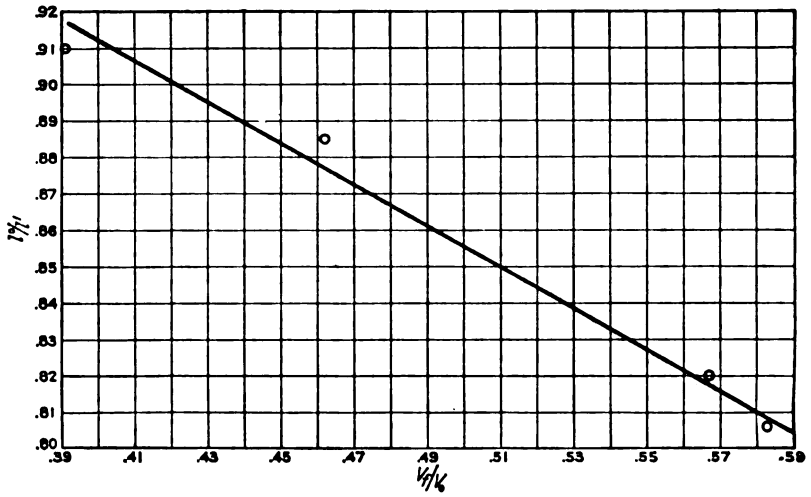

FIGURE 19.—Effect of rate of feed on apparent length of kiln for condition when the uniform depth of bed is greater than height of nose ring and nose ring is at discharge end of kiln; length of kiln, 7 feet; diameter of kiln, 6 inches; diameter of nose-ring opening, 4 inches; slope of kiln, 3°; speed of kiln, 2.5 r. p. m.

per 15 minutes and 19.6 minutes with feeds from 4,550 to 5,800 c. c. These were the volume limits used in the following calculations. By plotting $l°/l'$ v. v_f/v_o a curve approximating a straight line is produced. Although the points are not all on the "straight line," the error is never more than about 2 per cent. Table 36 gives the data and Figure 19 shows how closely the curve approaches a straight line.

TABLE 36.—*Effect of rate of feed on apparent length of kiln*

v_f	v_o	v_f/v_o	Ratio of $l°/l'$ 4.0-inch nose ring	4.5-inch nose ring	5.0-inch nose ring
15,300	39,100	0.391	0.91	0.941	1.0
18,050	39,100	.462	.885	.915	.985
22,200	39,100	.567	.820	.852	.930
22,800	39,100	.583	.806	.840	.919

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EQUATION OF CURVE

The equation of the curve was found to be:

$$l^\circ/l' = mv_f/v_o + 1.16$$

Plotting m as abscissa and d°/d' as ordinate, and taking the mean slope of the curve the equation $d^\circ/d' = m'(m) + b$ becomes $d^\circ/d' = -1.56\ m + 0.53$,
therefore

$$m = -0.64\ d^\circ/d' + 0.34.$$

Substituting for m in the equation $l^\circ/l' = mv_f/v_o + 1.16$, $l^\circ/l' = (0.34 - 0.64\ d^\circ/d')v_f/v_o + 1.16$

$$l' = \frac{l^\circ}{(0.34 - 0.64\ d^\circ/d')v_f/v_o + 1.16}$$

Adding the factor for revolutions per minute, the final equation then becomes:

$$t = \frac{1.77l\sqrt{\theta}}{pdn} \times \frac{1}{(0.34 - 0.64d^\circ/d')2.5v_f/nv_o + 1.16}$$

TESTS OF FORMULA

A few calculations were made from the data in Table 35 to see how well the formula holds. The calculated time was obtained by multiplying l'/l° by the time of passage at l°.

TABLE 37.—*Calculations using the formula for depth of bed greater than the height of the nose ring; pitch, 3°; n, 2.5*

Diameter of nose ring, inches	Feed per 15 minutes, c. c.	Time of passage, minutes		Diameter of nose ring, inches	Feed per 15 minutes, c. c.	Time of passage, minutes	
		Calculated	Observed			Calculated	Observed
4.0	4,400	22.2	21.8	4.5	5,445	22.8	22.8
4.0	5,295	23.8	23.3	5.0	3,190	19.1	19.4
4.5	4,460	21.1	21.2	5.0	5,670	20.1	21.2

CORRECTION FACTOR FOR CONDITION WHEN UNIFORM DEPTH OF BED IS GREATER THAN HEIGHT OF CONSTRICTION

The factor in the original formula for the condition when the uniform depth of bed of material is greater than the height of the constriction and the constriction is at the discharge end of the kiln is

$$\frac{1}{(0.34 - 0.64d^\circ/d')\frac{2.5v_f}{nv_o} + 1.16}$$

This is a correction factor for a special case. There must be a diaphragm, the diaphragm must be at the discharge end of the kiln,

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and the feed must be sufficient to give a uniform depth of bed greater than the height of the nose ring. For these reasons it is unnecessary for the formula to be of such a nature that $l' = l°$ when $d°/d' = 1$. This correction is simply an empirical one for a special condition.

The formula given is, of course, for kilns of the same dimensions as the laboratory kiln; that is, the ratio of diameter to length is 1 : 14. A general equation for kilns of different sizes can be worked out, but it is so complicated that a better way is to use one of the methods, preferably the last, described for the conditions where the uniform depth of bed is less than the height of the nose ring.

UNIFORM DEPTH OF BED OF MATERIAL GREATER THAN HEIGHT OF CONSTRICTION—CONSTRICTION NOT AT DISCHARGE END OF KILN

The equation for the condition when the depth of bed is greater than the height of the nose ring and the nose ring is at some other position than the discharge end of the kiln will now be solved. The data in Tables 24 and 25 show that once the uniform depth of bed is greater than the height of the constriction and the constriction is not directly at the discharge end of the kiln, the time of passage is virtually constant with increasing feed. Table 38 gives data on the effect of different-size constrictions on the time of passage.

TABLE 38.—*Uniform depth of bed greater than the height of the constriction, with constriction at some other position than the discharge end; n=2.5*

Diameter of nose-ring opening, inches	$d°$, inches	t' [1], minutes	$t°$ [2], minutes	$d°/d'$	$t'/t°$	Feed per 15 minutes, c.c.
3.0	6.0	24.1	19.9	2.0	1.21	6,150
3.75	6.0	21.4	18.7	1.6	1.143	2,280
4.0	6.0	21.35	18.8	1.5	1.135	3,400
4.5	6.0	19.6	18.8	1.33	1.041	3,180
5.0	6.0	19.1	18.8	1.20	.019	3,290

[1] t' = time of passage with constriction.
[2] $t°$ = time of passage without constriction for same feed as t'.

PLOTTING OF CURVE

By plotting $d°/d'$ against $t'/t°$ and drawing the average straight line which passes through the origin (where $d°/d' = 1$, and $t'/t° = 1$), the equation becomes

$$d°/d' = 4.76\ (t'/t°) - 3.76, \text{ or}$$

$$t' = \frac{t°\,(d°/d' + 3.76)}{4.76}.$$

When no constriction is present, when

$$d° = d',\ t' = t°.$$

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TESTS OF ACCURACY OF FORMULA

Some calculations were made to test the accuracy of the formula. Table 39 gives the results.

TABLE 39.—*Tests of formula*

$d°/d'$	$t°$, minutes	t', minutes		$d°/d'$	$t°$, minutes	t', minutes	
		Calculated	Observed			Calculated	Observed
2.0	19.9	24.1	24.1	1.33	18.8	20.05	19.6
1.60	18.7	21.05	21.4	1.20	18.8	19.6	19.1
1.50	18.8	20.8	21.35				

APPLICATION OF FORMULA TO DIFFERENT RATIOS OF DIAMETER TO LENGTH

This formula can be converted quite easily into a general formula for different ratios of diameter to length of kiln. The general form of the equation is

$$d°/d' = at'/t° + b$$

The two constants a and b are the only factors in the formula which change with different ratios of diameter to length of the kiln. Table 40 contains the values of a and b for different ratios of diameter to length.

TABLE 40.—*Values of constants a and b for different ratios of diameter to length*

l/d	a	b	l/d	a	b
5	1.70	−0.70	10	3.33	−2.33
7	2.53	−1.53	12	4.00	−3.00
9	2.90	−1.90	14	4.76	−3.76

By plotting l/d against a, the equation is found to be

$$a = 0.3l/d + 0.195$$

and plotting l/d against b, the equation is

$$b = 0.8 - 0.3l/d$$

Substituting these values in the general equation gives

$$t' = t° \times \frac{d°/d' - (0.8 - 0.3l/d)}{0.3l/d + 0.195}$$

The complete equation for the case in which the uniform depth of bed is greater than the height of the nose ring, and the nose ring is not at the discharge end of the kiln is

$$t = \frac{1.77l\sqrt{\theta}}{pdn} \times \frac{d°/d' - (0.8 - 0.3l/d)}{0.3l/d + 0.195}.$$

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The "factor" in the original equation for this special case is

$$\frac{d^\circ/d' - (0.8 - 0.3l/d)}{0.3l/d + 0.195}.$$

If this general formula is not used, equations for kilns of the ratio of diameter to length other than 1:14 can be derived by one of the methods described for calculations where the uniform bed is less than the height of the constriction.

TESTS OF ACCURACY OF CORRECTION FACTOR

Some calculations made to test the accuracy of this correction factor are tabulated in Table 41.

TABLE 41.—*Values derived from the general correction factor for kilns in which the uniform depth of bed is greater than the height of the constriction and the latter is not at the discharge end of the kiln*

[Ratio of $d:l = 1:14$; slope = 3°]

d°/d'	t°, minutes	t', minutes		d°/d'	t°, minutes	t', minutes	
		Calculated	Observed			Calculated	Observed
2.0	19.9	24.3	24.1	1.33	18.8	20.2	19.6
1.60	18.7	21.2	21.4	1.20	18.8	19.6	19.1
1.50	18.8	20.8	21.35				

SUMMARY

There have now been presented formulas for four different kiln conditions, as follows:

1. The formula for a simple kiln with no obstructions or constrictions may be expressed by the following formula:

$$t = \frac{1.77l\sqrt{\theta}}{pdn}.$$

2. For the condition where the uniform depth of bed of material in the kiln is less than the height of the constriction the following formula has been presented:

$$t = \frac{1.77l\sqrt{\theta}}{pdn} \times e^{\left[(0.12l/d - 3.86)\log \frac{2.5v_f}{nv_o} + e^{1.3 - 0.11l/d} - 1\right]\frac{h}{d^\circ - 2h}}.$$

3. When the uniform depth of bed of material is greater than the height of the constriction and the constriction is at the discharge end of the kiln, the equation is

$$t = \frac{1.77l\sqrt{\theta}}{pdn} \times \frac{1}{(0.34 - 0.64d^\circ/d')\,\frac{2.5v_f}{nv_o} + 1.16}.$$

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4. When the uniform depth of bed of material is greater than the height of the constriction and the latter is at a position other than the discharge end of the kiln, the following is the equation:

$$t = \frac{1.77l\sqrt{\theta}}{pdn} \times \frac{d^\circ/d' - (0.8 - 0.3l/d)}{0.3l/d + 0.195}.$$

Equation 3 holds when the ratio of diameter of the kiln to the length of the kiln is 1:14. For other ratios methods of calculation have been given. The formulas are for constrictions of narrow width. The correction factors, as determined in the laboratory kiln, are probably accurate to better than 5 per cent. (See p. 1 for explanation of symbols.)

COMMERCIAL KILNS AND THE KILN EQUATION

This investigation of the passage of solid particles through rotary kilns was conducted under what might be termed "ideal conditions." No chemical change occurred, and the angle of repose remained constant during the passage of the material through the kiln. Although this is true, the simple straight-kiln formula has been found to work very well in all commercial kilns that were available for study. The authors visited a number of cement kilns and two coal driers and took measurements. The calculated time of passage in one cement kiln 125 feet long was 73 minutes; from observations made during the manufacture of oil-well cement, where certain ingredients were added at a definite time and analyses were made of the clinker at given intervals, it was found that the time of passage actually ranged from 60 to 70 minutes. For a clinker cooler 60 feet long the calculated time of passage was 26 minutes and the observed time 25 minutes. For a single-shell externally heated coal drier the calculated time of passage was 21.4 minutes. The kiln was not running when seen, but figures available at the plant showed that the time of passage was a little less than 25 minutes.

For a Ruggles-Cowles drier, with an external shell 7½ feet in diameter and a space of 1½ feet between the inner and outer shell, the time observed was about 38 minutes. Calculating this on the basis of a simple 7½-foot kiln gives the time of passage as 12½ minutes. However, by calculating as a 1½-foot kiln, the difference between the shells, the calculated time was 39 minutes. No more kilns of this type were available for checking this observation; so it is mentioned for what it is worth.

Figure 20 is a nomographic chart for the unobstructed kiln equation. It is a simple type of alignment chart; its construction and use are the same as those of any ordinary chart of this nature. A discussion of the construction of nomographic charts is out of place

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here, and anyone interested is referred to such standard texts as Lipka, "Graphical and Mechanical Computation," or Peddle, "The Construction of Graphical Charts." The method of using the chart is indicated by the "Key" and the illustrative example.

SUMMARY

Empirical formulas have been presented which enable one to calculate the time of passage of solid particles through rotary cylindrical kilns. Correction factors for nose rings or constrictions have been added. For a given kiln the formulas, together with the apparent density of the material to be used, the rate of feed, and simple formulas of trigonometry and geometry, enable one to calculate other kiln factors, such as volume of material in the kiln, depth of bed, length of chord, and capacity. In the simple kiln equation there are six variables; if any five are known, the other can be determined.

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